Lesson 5

Solve Multi-Step Equations

ANSWER KEY

Real-World Link

Lacrosse Coach Everly wants to order uniform shirts for all the players p on her women's lacrosse team. Each shirt costs $20. There is an additional cost d for a player to put her name on the shirt. Use the steps below to write an equation for the total cost c if every player on the team orders a shirt with her name on it.

1. Circle the variables above and underline what they represent.

2. Write an expression that represents the cost of one shirt with a player's name on it.

$\square + \square$

cost of shirt + cost of name

3. Use the expression to write an equation that can be used to find the total cost if every player on the team orders a shirt with her name on it.

$\square\left(\square + \square\right) = c$

number of players (cost of shirt + cost of name) = total cost

4. Suppose the total cost for 15 players to buy shirts is $420. Write an equation to show the total cost of the shirts if all of the players put their names on the shirts.

Essential Question

WHAT is equaivalence?

Vocabulary

null set
identity

Math Symbols

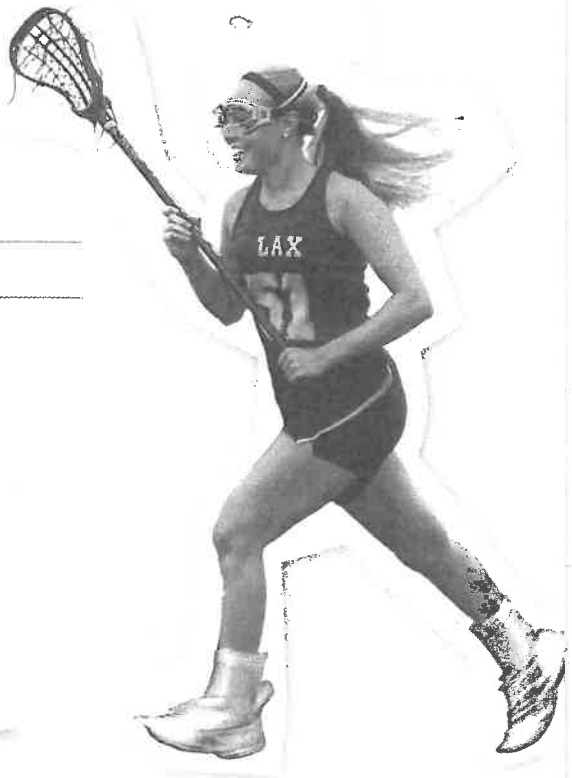
$\varnothing$ null set
$\{\}$ empty set

Common Core State Standards

Content Standards
8.EE.7, 8.EE.7a, 8.EE.7b

Mathematical Practices
1, 2, 3, 4

where did it go?



Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Solve Multi-Step Equations

Some equations contain expressions with grouping symbols. To solve these equations, first expand the expression using the Distributive Property. Then collect like terms if needed, and solve the equation using the Properties of Equality.

Example

Tutor

1. **Solve $15(20 + d) = 420$.**

$15(20 + d) =$	420	Write the equation.
$300 + 15d =$	420	Distributive Property
-300	$= -300$	Subtraction Property of Equality
$15d =$	120	Simplify.
$\frac{15d}{15} =$	$\frac{120}{15}$	Division Property of Equality
$d =$	8	Simplify.

Show your work

Got it? **Do these problems to find out.**

a. $-3(9 + x) = 33$

$-27 - 3x = 33$
$+27 \quad +27$
$\frac{-3x}{-3} = \frac{60}{-3}$ $x = -20$

a. $x = -20$

b. $5(a - 7) = 25$

$5a - 35 = 25$
$+35 \quad +35$
$\frac{5a}{5} = \frac{60}{5}$ $a = 12$

b. $a = 12$

Key Concept — Number of Solutions

	Null Set	One Solution	Identity
Words	no solution	one solution	infinitely many solutions
Symbols	$a = b$	$x = a$	$a = a$
Example	$3x + 4 = 3x$ $4 = 0$ Since $4 \neq 0$, there is no solution.	$2x = 20$ $x = 10$	$4x + 2 = 4x + 2$ $2 = 2$ Since $2 = 2$, the solution is all numbers.

Some equations have no solution. When this occurs, the solution is the **null set** or empty set and is shown by the symbol $\varnothing$ or $\{\}$. Other equations may have every number as their solution. An equation that is true for every value of the variable is called an **identity**.

Examples

Tutor

2. **Solve $6(x - 3) + 10 = 2(3x - 4)$.**

$6(x - 3) + 10 = 2(3x - 4)$	Write the equation.
$6x - 18 + 10 = 6x - 8$	Distributive Property
$6x - 8 = 6x - 8$	Collect like terms.
$+8 = +8$	Addition Property of Equality
$6x = 6x$	Simplify.
$\frac{6x}{6} = \frac{6x}{6}$	Division Property of Equality
$x = x$	Simplify.

The statement $x = x$ is *always* true. The equation is an identity and the solution set is all numbers.

Check $6(x - 3) + 10 = 2(3x - 4)$	Write the original equation.
$6(5 - 3) + 10 \stackrel{?}{=} 2[3(5) - 4]$	Substitute any value for x.
$6(2) + 10 \stackrel{?}{=} 2(15 - 4)$	Simplify.
$22 = 22$ ✓	

3. **Solve $8(4 - 2x) = 4(3 - 5x) + 4x$.**

$8(4 - 2x) = 4(3 - 5x) + 4x$	Write the equation.
$32 - 16x = 12 - 20x + 4x$	Distributive Property
$32 - 16x = 12 - 16x$	Collect like terms.
$+16x = +16x$	Addition Property of Equality
$32 = 12$	Simplify.

The statement $32 = 12$ is *never* true. The equation has no solution and the solution set is $\varnothing$.

Check $8(4 - 2x) = 4(3 - 5x) + 4x$	Write the equation.
$8[4 - 2(2)] \stackrel{?}{=} 4[3 - 5(2)] + 4(2)$	Substitute any value for x.
$8(0) \stackrel{?}{=} 4(-7) + 8$	Simplify.
$0 \neq -20$ ✓	Since $0 \neq -20$, the equation has no solution.

STOP and Reflect

How do you know if the solution $5 = 0$ indicates no solution, one solution, or infinitely many solutions?

Got it? **Do these problems to find out.**

c. $3(6 - 4x) = -2(6x - 9)$

$18 - 12x = -12x + 18$
$+12x \quad +12x$
$18 = 18$

identity = unlimited solutions

d. $2(3x + 5) = 5(2x - 4) - 4x$

$6x + 10 = 10x - 20 - 4x$
$6x + 10 = 6x - 20$
$-6x \quad -6x$
$10 = -20$

Null Set
No Solution

Show your work.

c. Unlimited solutions

d. No Solution

Example

Watch Tutor

4. **At the fair, Hunter bought 3 snacks and 10 ride tickets. Each ride ticket costs $1.50 less than a snack. If he spent a total of $24.00, what was the cost of each snack?**

Write an equation to represent the problem.

$3s + 10(s - 1.5) = 24$	Write the equation.
$3s + 10s - 15 = 24$	Distributive Property
$13s - 15 = 24$	Collect like terms.
$+15 = +15$	Addition Property of Equality
$13s = 39$	Simplify.
$\frac{13s}{13} = \frac{39}{13}$	Division Property of Equality
$s = 3$	Simplify.

So, the cost of each snack was $3.

Guided Practice

Check

Solve each equation. Check your solution. (Examples 1–3)

1. $-8(w - 6) = 32$

Show your work.

$-8w + 48 = 32$
$-48 \quad -48$
$\frac{-8w}{-8} = \frac{-16}{-8}$
$w = 8$

2. $8z - 22 = 3(3z + 11) - z$

$8z - 22 = 9z + 33 - z$
$8z - 22 = 8z + 33$
$-8z \quad -8z$
$-22 = 33$

No solution

$J = R + 10$
$S = 2(R + 10)$
$R = R$

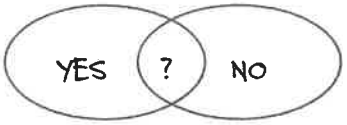
3. Mr. Richards' class is holding a canned food drive for charity. Juliet collected 10 more cans than Rosana. Santiago collected twice as many cans as Juliet. If they collected 130 cans altogether, how many cans did Juliet collect? (Example 4) Rosana collected 25 so Julie collected 35

$130 = r + 10 + 2(r + 10) + r$
$130 = r + 10 + 2r + 20 + r$
$130 = 4r + 30$
$-30 \quad -30$
$\frac{100}{4} = \frac{4r}{4}$ $r = 25$

4. **Building on the Essential Question** How many possible solutions are there to a linear equation in one variable? Describe each one.

Rate Yourself!

Are you ready to move on? Shade the section that applies.



For more help, go online to access a Personal Tutor.

Tutor

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Go online for Step-by-Step Solutions eHelp

Solve each equation. Check your solution. (Examples 1–3)

1. $-12(k + 4) = 60$

$-12k - 48 = 60$
$+48 \quad +48$

Show your work.

$\frac{-12k}{-12} = \frac{108}{-12}$

$k = -9$

2. $8(3a + 6) = 9(2a - 4)$

$24a + 48 = 18a - 36$
$+36 \quad +36$

$24a + 84 = 18a$
$-24a \quad -24a$

$\frac{84}{6} = \frac{-6a}{6}$

$-14 = a$

3. $\frac{1}{3}h - 4\left(\frac{2}{3}h - 3\right) = \frac{2}{3}h - 6$

$\frac{1}{3}h - \frac{8}{3}h + 12 = \frac{2}{3}h - 6$

$-\frac{7}{3}h + 12 = \frac{2}{3}h - 6$
$+6 \quad +6$

$-\frac{7}{3}h + 18 = \frac{2}{3}h$
$+\frac{7}{3}h \quad +\frac{7}{3}h$

$18 = \frac{9}{3}h$

$\frac{18}{3} = \frac{3h}{3}$

$h = 6$

4. $8(c - 9) = 6(2c - 12) - 4c$

$8c - 72 = 12c - 72 - 4c$

$8c - 72 = 8c - 72$
$+72 \quad +72$

$8c = 8c$

Unlimited Solutions

Copy and Solve Solve each equation. Show your work on a separate piece of paper. (Examples 2 and 3)

5. $-10y + 18 = -3(5y - 7) + 5y$

6. $8(t + 2) - 3(t - 4) = 6(t - 7) + 8$

7. $4(5 + 2x) - 5 = 3(3x + 7)$

8. $6(2x - 8) + 3 = 15$

See last page for answers

9. The school has budgeted $2,000 for an end-of-year party at the local park. The cost to rent the park shelter is $150. How much can the student council spend per student on food if each of the 225 students receives a $3.50 gift? (Example 4) $4.72 per student

$2{,}000 = 225x + 225(3.50) + 150$
$2{,}000 = 225x + 787.50 + 150$
$2{,}000 = 225x + 937.50$
$-937.50 \quad -937.50$

$\frac{1{,}062.50}{225} = \frac{225x}{225}$

$4.72 = x$

10. MP **Reason Abstractly** The table shows the number of students in each homeroom.

Teacher	Number of Students
Mr. Boggs	b
Mr. Hamilton	$1.5(b + 2)$
Ms. Simpson	15
Mrs. Walton	$2b - 9$

a. Write an equation to find the number of students in Mr. Boggs' homeroom if the total number of students is 90. $b + 1.5(b + 2) + 15 + 2b - 9 = 90$

b. Solve the equation from part **a** to find the number of students in Mr. Boggs' homeroom.

$b + 1.5b + 3 + 15 + 2b - 9 = 90$

$4.5b + 9 = 90$
$-9 \quad -9$

$\frac{4.5b}{4.5} = \frac{81}{4.5}$

$b = 18$

11. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.

a. Write an equation that can be used to determine the number of text messages Jacob and Roberto can send for their plans to cost the same.

$0.15t + 20 = 0.10t + 30$

b. Solve the equation from part **a** to find the number of text messages each person can send for their costs to be the same.

200 text messages

$0.15t + 20 = 0.10t + 30$
$-20 \quad -20$
$0.15t = 0.10t + 10$
$0.10t \quad -0.10t$
$\frac{0.05t}{0.05} = \frac{10}{0.05}$
$t = 200$

H.O.T. Problems Higher Order Thinking

12. **Reason Inductively** Does a multi-step equation *always*, *sometimes*, or *never* have a solution? Explain your reasoning.

Sometimes, ex: a problem like $2w - 8 = 2w + 5$ has no solution

13. **Persevere with Problems** The perimeter of a rectangle is $8(2x + 1)$ inches. The length of the sides of the rectangle are $3x + 4$ inches and $4x + 3$ inches. Write and solve an equation to find the length of each side of the rectangle.

$3(3) + 4 = 13$
$4(3) + 3 = 15$

13 and 15 inches

$2(3x+4) + 2(4x+3) = 8(2x+1)$
$6x + 8 + 8x + 6 = 16x + 8$
$14x + 14 = 16x + 8$
$-14x \quad -14x$
$\frac{14}{-8} = \frac{2x}{-8}$
$\frac{6}{2} = \frac{2x}{2}$
$3 = x$

14. **Model with Mathematics** Write a real-world problem that can be solved using the Distributive Property. Then write and solve an equation for your real-world situation.

may vary

Name _____ My Homework _____

Extra Practice

Solve each equation. Check your solution.

15. $9(j - 4) = 81$

Homework Help

$9j - 36 = 81$
$+36 = +36$
$9j = 117$
$\frac{9j}{9} = \frac{117}{9}$
$j = 13$

16. $8(4q - 5) - 7q = 5(5q - 8)$

$32q - 40 - 7q = 25q - 40$
$25q - 40 = 25q - 40$
$-25q = -25q$
$-40 = -40$
The solution set is all numbers.

17. $\frac{1}{2}r + 2\left(\frac{3}{4}r - 1\right) = \frac{1}{4}r + 6$

$\frac{2}{4}r + \frac{6}{4}r - 2 = \frac{1}{4}r + 6$
$\frac{8}{4}r - 2 = \frac{1}{4}r + 6$
$-\frac{1}{4}r \quad -\frac{1}{4}r$
$\frac{7}{4}r - \frac{2}{+2} = \frac{6}{+2}$
$\frac{4}{7} \cdot \frac{7}{4}r = \frac{8}{1} \cdot \frac{4}{7}$
$r = \frac{32}{7} = 4\frac{4}{7}$

18. $-5(3m + 6) = -3(4m - 2)$

$-15m - 30 = -12m + 6$
$+12m \quad +12m$
$-3m - 30 = 6$
$+30 \quad +30$
$\frac{-3m}{-3} = \frac{36}{-3}$
$m = -12$

19. $-7(k + 9) = 9(k - 5) - 14k$

$-7k - 63 = 9k - 45 - 14k$
$-7k - 63 = -5k - 45$
$+7k \quad +7k$
$-63 = 2k - 45$
$+45 \quad +45$
$\frac{-18}{2} = \frac{2k}{2}$
$-9 = k$

20. $10p - 2(3p - 6) = 4(3p - 6) - 8p$

$10p - 6p + 12 = 12p - 24 - 8p$
$4p + 12 = 4p - 24$
$-4p \quad -4p$
$12 \neq -24$
no solution

21. $12(x + 3) = 4(2x + 9) + 4x$

$12x + 36 = 8x + 36 + 4x$
$12x + 36 = 12x + 36$
infinite solutions

22. $0.2(x + 50) - 6 = 0.4(3x + 20)$

$0.2x + 10 - 6 = 1.2x + 8$
$-.2x \quad -.2x$
$4 = x + 8$
$-8 \quad -8$
$-4 = x$

23. MP **Identify Structure** Give an example of a multi-step equation for each of the following solutions. may vary...

a. all numbers Ex: $3x + 5 = 3x - 2 + 7$

b. null set Ex: $2(x - 1) = 2x + 2$

Power Up! Common Core Test Practice

24. The table shows expressions to represent the number of students involved in different activities. The number of students involved in sports and student council is equal to the number of students involved in band and drama club. Model the situation with an equation. Select the correct expression for each box. Then solve the equation.

Activity	Number of Students
band	$3n - 2$
drama club	$2(2n + 1)$
sports	$5n + 7$
student council	n

$\boxed{5n+7} + \boxed{n} = \boxed{3n-2} + \boxed{2(2n+1)}$

$5n + 7$	$3n - 2$
n	$2(2n + 1)$

$n = \boxed{7}$

$5n+7+n=3n-2+2(2n+1)$
$6n+7=3n-2+4n+2$
$6n+7=7n$
$-6n \quad -6n$
$7=n$

How many students participate in each activity?

sports: $\boxed{42}$ $5(7)+7$

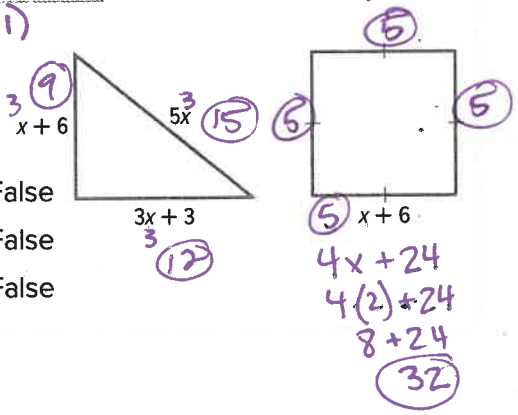
band: $\boxed{19}$ $3(7)-2$

student council: $\boxed{7}$

drama club: $\boxed{30}$ $2(2(7)+1)$

6. $x+6+5x+3x+3=36$
$9x+9=36$
$-9 \quad -9$
$\frac{9x}{9} = \frac{27}{9}$
$x = 3$

25. The figures below have the same perimeter. Determine if each statement is true or false.



9 15 12

$4x+24$
$4(2)+24$
$8+24$
32

a. The value of x is 2. True False

b. The perimeter of the triangle is 36 units. True False

c. The perimeter of the square is 32 units. True False

$4x+36=32$
$-36 \quad -36$
$\frac{4x}{4} = \frac{-4}{4}$
$x=-1$

Common Core Spiral Review

Solve each inequality. Graph the solution set on a number line. 7.EE.4b

26. $a - 5 < 2$
$+5 \quad +5$
$a < 7$

27. $x - 9 \geq -12$ $x \geq -3$
$+9 \quad +9$

28. $\frac{5y}{5} \leq \frac{-30}{5}$
$y \leq -6$

29. $-\frac{n}{4} > -2$ $-4 \quad n < 8$
-4

you might not know this... yet! it's okay!

Pg 157

#5.) $-10y+18=-3(5y-7)+5y$

$-10y+18=-15y+21+5y$

$-10y+18=-10y+21$

$+10y \qquad +10y$

$18 \neq 21$

no solution

#6.) $8(t+2)-3(t-4)=6(t-7)+8$

$8t+16-3t+12=6t-42+8$

$5t+28=6t-34$

$-5t \qquad -5t$

$28=t-34$

$+34 \qquad +34$

$64=t$

#7.) $4(5+2x)-5=3(3x+7)$

$20+8x-5=9x+21$

$15+8x=9x+21$

$-15 \qquad -15$

$8x=9x+6$

$-9x \quad -9x$

$\frac{-x}{-1}=\frac{6}{-1}$

$x=-6$

#8.) $6(2x-8)+3=15$

$12x-48+3=15$

$12x-45=15$

$+45 \quad +45$

$\frac{12x}{12}=\frac{60}{12}$

$x=5$