

Solve Equations with Rational Coefficients

Vocabulary Start-Up



Two numbers with a product of 1, such as $\frac{3}{4}$ and $\frac{4}{3}$, are called reciprocals or **multiplicative inverses**.

Complete the graphic organizer.

<p>Define It</p> <p>two fractions that multiply to give one as the answer</p>	<p>Describe It</p> <p>the numerator and denominator of a fraction switch places</p>
<p>List Some Examples</p> <p>$\frac{2}{3}$ and $\frac{3}{2}$ or $-\frac{1}{2}$ and $\frac{2}{1}$</p>	<p>List Some Nonexamples</p> <p>$2\frac{1}{2}$ and $\frac{7}{3}$ or $\frac{1}{2}$ and $-\frac{2}{1}$</p>

multiplicative inverses



Essential Question

WHAT is equivalence?



Vocabulary

multiplicative inverse
coefficient



Common Core State Standards

Content Standards
8.EE.7, 8.EE.7a, 8.EE.7b

Mathematical Practices
1, 3, 4, 7

Describe how a multiplicative inverse is used in division of fractions.

When you divide fractions, multiply the dividend by the multiplicative inverse of the divisor.



Real-World Link

How can the action of the motorcyclist in the photo help you remember what the multiplicative inverse is?

The motorcyclist is upside down.



Which **MP** Mathematical Practices did you use?

Shade the circle(s) that applies.

- | | |
|--|---|
| <input type="checkbox"/> 1 Persevere with Problems | <input type="checkbox"/> 5 Use Math Tools |
| <input type="checkbox"/> 2 Reason Abstractly | <input type="checkbox"/> 6 Attend to Precision |
| <input type="checkbox"/> 3 Construct an Argument | <input type="checkbox"/> 7 Make Use of Structure |
| <input type="checkbox"/> 4 Model with Mathematics | <input type="checkbox"/> 8 Use Repeated Reasoning |

Key Concept

Inverse Property of Multiplication

Work Zone



and Reflect

What is the multiplicative inverse of $\frac{3}{2}$?

$-\frac{2}{3}$

Words The product of a number and its multiplicative inverse is 1.

Numbers $\frac{7}{8} \times \frac{8}{7} = 1$ $-\frac{3}{2} \times -\frac{2}{3} = 1$

Symbols $\frac{a}{b} \cdot \frac{b}{a} = 1$, where a and $b \neq 0$

The numerical factor of a term that contains a variable is called the **coefficient** of the variable.



In the equation $\frac{3}{4}c = 18$, the coefficient of c is a rational number. To solve an equation when the coefficient is a fraction, multiply each side by the multiplicative inverse of the fraction.

Example

Tutor

1. Solve $\frac{3}{4}c = 18$. Check your solution.

$\frac{3}{4}c = 18$ Write the equation.

$\left(\frac{4}{3}\right) \cdot \frac{3}{4}c = \left(\frac{4}{3}\right) \cdot 18$ Multiply each side by the multiplicative inverse of $\frac{3}{4}$, $\frac{4}{3}$.

$\frac{1}{\cancel{3}} \cdot \frac{1}{\cancel{4}}c = \frac{4}{\cancel{3}} \cdot \frac{18}{1}$ Write 18 as $\frac{18}{1}$. Divide by common factors.

$c = 24$ Simplify.

Check $\frac{3}{4}c = 18$ Write the original equation.

$\frac{3}{4}(24) \stackrel{?}{=} 18$ Replace c with 24.

$\frac{3}{\cancel{4}} \left(\frac{24}{\cancel{1}}\right) \stackrel{?}{=} 18$ Write 24 as $\frac{24}{1}$. Divide by common factors.

$18 = 18$ ✓ This sentence is true.

a. 60

b. -18

c. 9

d. 28

Got it? Do these problems to find out.

a. $\frac{1}{5}x = 12$

b. $-\frac{2}{9}d = 4$

c. $15 = \frac{5}{3}n$

d. $-24 = -\frac{6}{7}p$

Example



2. Solve $1\frac{1}{2}s = 16\frac{1}{2}$. Check your solution.

$$1\frac{1}{2}s = 16\frac{1}{2}$$

Write the equation.

$$\frac{3}{2}s = \frac{33}{2}$$

Rename $1\frac{1}{2}$ as $\frac{3}{2}$ and $16\frac{1}{2}$ as $\frac{33}{2}$.

$$\left(\frac{2}{3}\right) \cdot \frac{3}{2}s = \left(\frac{2}{3}\right) \cdot \frac{33}{2}$$

Multiply each side by the multiplicative inverse of $\frac{3}{2}$, $\frac{2}{3}$.

$$\frac{1\cancel{2}}{\cancel{2}1} \cdot \frac{1\cancel{3}}{\cancel{3}1}s = \frac{1\cancel{2}}{\cancel{2}1} \cdot \frac{11}{1}$$

Divide by common factors.

$$s = 11$$

Simplify.

Got it? Do these problems to find out.

d. $4\frac{1}{6} = 3\frac{1}{3}c$

e. $-9\frac{5}{8}w = 108$

f. $1\frac{7}{8}y = 4\frac{1}{2}$

Show your work.

d. 14

e. -11 $\frac{17}{7}$

f. 2 $\frac{2}{5}$

Solve Equations with Decimal Coefficients

In the equation $3.15 = 0.45n$ the coefficient of n is a decimal. To solve an equation with a decimal coefficient, divide each side of the equation by the coefficient.

Example



3. Solve $3.15 = 0.45n$. Check your solution.

$$3.15 = 0.45n$$

Write the equation.

$$\frac{3.15}{0.45} = \frac{0.45n}{0.45}$$

Division Property of Equality

$$7 = n$$

Simplify.

Check $3.15 = 0.45n$

Write the original equation.

$$3.15 = 0.45(7)$$

Replace n with 7.

$$3.15 = 3.15 \checkmark$$

The sentence is true.

Quick Review

Division

$$\begin{array}{r} 7 \\ 0.45 \overline{)3.15} \\ \underline{-3.15} \\ 0 \end{array}$$

g. 7

h. -1.5

i. 2.3

Got it? Do these problems to find out.

g. $4.9 = 0.7t$

h. $-1.4m = 2.1$

i. $-5.6k = -12.88$



Example

Tutor



Quick Review

To write a percent as a decimal, move the decimal point two places to the left. Add zeros, if necessary. For example, $3\% = 0.03$ and $75\% = 0.75$.

- 4.** Latoya's softball team won 75%, or 18, of its games. Define a variable. Then write and solve an equation to determine the number of games the team played.

Latoya's softball team won 18 games, which was 75% of the games played. Let n represent the number of games played. Write and solve an equation.

$$0.75n = 18 \quad \text{Write the equation. Write 75\% as 0.75.}$$

$$\frac{0.75n}{0.75} = \frac{18}{0.75} \quad \text{Division Property of Equality}$$

$$n = 24 \quad \text{Simplify.}$$

Latoya's softball team played 24 games.

Guided Practice

Check



Solve each equation. Check your solution. (Examples 1–3)

1. $60 = \frac{3}{4}p$

80

Show your work.

2. $-\frac{27}{25}x = -\frac{9}{5}$

$1\frac{2}{3}$

3. $-2.7t = 810$

-300

- 4.** Paula has read 70% of the total pages in a book she is reading for English class. Paula has read 84 pages. Define a variable. Then write and solve an equation to determine how many pages are in the book. (Example 4)

$p = \text{total pages in book}$

$$0.7p = 84$$

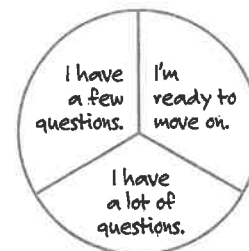
120 pages

- 5.** **Building on the Essential Question** How is the multiplicative inverse used to solve an equation that has a rational coefficient?

To solve an equation with a coefficient that is a fraction, multiply each side of the equation by the multiplicative inverse of the fraction.

Rate Yourself!

Are you ready to move on?
Shade the section that applies.



For more help, go online to access a Personal Tutor.

Tutor



Independent Practice

Go online for Step-by-Step Solutions

eHelp



Solve each equation. Check your solution. (Examples 1–3)

1. $6 = \frac{1}{12}v$

(72)

Show your work

2. $-\frac{2}{3}w = 60$

(-90)

3. $-\frac{7}{8}k = -21$

(24)

4. $9.6 = 1.2b$

(8)

5. $0.75a = -9$

(-12)

6. $-413.4 = -15.9n$

(26)

7. $3\frac{1}{10}s = 6\frac{1}{5}$

(2)

8. $2\frac{2}{9} = -\frac{4}{5}m$

 (-2 $\frac{7}{9}$)

9. $-2\frac{4}{5} = -3\frac{1}{2}n$

 ($\frac{4}{5}$)

Define a variable. Then write and solve an equation for each situation. (Example 4)

10. The Parker family drove a total of 180 miles on their road trip. This distance is 1.5 times the distance they drove on the first day. How many miles did the Parker family drive on the first day?

$d =$ miles traveled on first day

$$1.5d = 180$$

120 miles

11. José correctly answered 80% of the questions on a language arts quiz. If he answered 16 questions correctly, how many questions were on the language arts quiz?

$q =$ total questions

$$0.8q = 16$$

20 questions

12. **Financial Literacy** Demetrius deposited 60% of his paycheck into his savings account. What was the amount of his paycheck?

$a =$ amount in paycheck

$$0.60a = 41.67$$

\$69.45

Savings Deposit Slip

Demetrius Matthews

Name

Amount Deposited

\$41.67

13. **MP Identify Structure** Suppose the numbers, $\frac{1}{3}$, 0.2 , -5 , $-\frac{1}{2}$, are each coefficients in separate equations. Choose whether you would solve the equation by multiplying each side by the multiplicative inverse of the coefficient or by dividing each side by the coefficient. Write the number in the appropriate space.

Multiplicative
Inverse
 $\frac{1}{3}$ $-\frac{1}{2}$

Division
 0.2 -5

H.O.T. Problems Higher Order Thinking

14. **MP Model with Mathematics** Write a real-world problem that can be represented by the equation $\frac{3}{4}c = 21$.

Ex: $\frac{3}{4}$ of the students in my homeroom study spanish. 21 students study spanish. How many students are in my homeroom?

- MP Persevere with Problems** Determine whether each statement is true or false. Explain your reasoning.

15. The product of a fraction and its multiplicative inverse is 1.

true, the product of $\frac{3}{4}$ and $\frac{4}{3}$ is $\frac{12}{12}$ which simplifies to 1.

16. To solve an equation with a coefficient that is a fraction, divide each side of the equation by the reciprocal of the fraction.

false, you would multiply, not divide, by the reciprocal of the fraction.

17. **MP Reason Inductively** Complete the statement: If $10 = \frac{1}{5}x$, then

$x + 3 =$. Explain your reasoning. 53; since $10 = \frac{1}{5}x$ then $x = 50$ and $x + 3 = 53$

18. **MP Justify Conclusions** Suppose your friend says he can solve $3x = 15$ by using the Multiplication Property of Equality. Is he correct? Justify your response.

Yes, he can multiply each side of the equation by $\frac{1}{3}$ instead of dividing by 3.

Extra Practice

Solve each equation. Check your solution.

19. $\frac{1}{2} = \frac{2}{5}z$

Homework Help

$$\begin{aligned} \frac{5}{2} \cdot \frac{1}{2} &= \frac{5}{2} \cdot \frac{2}{5}z \\ \frac{5}{4} &= 1z \\ 1\frac{1}{4} &= z \end{aligned}$$

20. $-\frac{3}{4}t = 5$

$$-6\frac{2}{3}$$

21. $-\frac{2}{9}g = -\frac{7}{9}$

$$3\frac{1}{2}$$

22. $0.6w = 0.48$

$$0.8$$

23. $-226.8 = 21.6y$

$$-10.5$$

24. $-30 = 1.25c$

$$-24$$

25. $1\frac{1}{2}x = 9\frac{9}{20}$

$$6\frac{3}{10}$$

26. $-12\frac{2}{3} = -1\frac{1}{9}y$

$$11\frac{2}{5}$$

27. $1\frac{5}{7} = 1\frac{13}{14}a$

$$\frac{8}{9}$$

28. One third of the bagels in a bakery are sesame bagels. There are 72 sesame bagels. Define a variable. Then write and solve an equation to find how many bagels there were in the bakery.

$b = \text{total \# of bagels}$

$$\frac{1}{3}b = 72$$

$$b = 216$$

216 bagels

29. **MP Find the Error** Sarah is solving the equation $-\frac{7}{8}x = 24$.

Circle her mistake and correct it.

$$\left(-\frac{8}{7}\right) \cdot \frac{7}{8}x = 24 \left(-\frac{8}{7}\right)$$

$$x = -27\frac{3}{7}$$

$$\begin{aligned} -\frac{7}{8}x &= 24 \\ \left(\frac{7}{8}\right) \cdot \frac{7}{8}x &= 24 \left(\frac{7}{8}\right) \\ x &= 21 \end{aligned}$$





Power Up! Common Core Test Practice

30. Select the correct equation for each situation. Then solve each problem.

- a. The Jansen family drove a total of 240 miles on their road trip. This distance is 5 times the distance they drove on the first day. How many miles did the family drive on the first day?

$\frac{x}{5} = 240$	$5x = 240$
$\frac{x}{240} = 0.5$	$0.05x = 240$
$0.5x = 240$	$\frac{5}{x} = 240$

Equation: $5x = 240$ Solution: 48 miles

- b. There are 240 students in Misty's school. This is 5% of the total students in the school district. How many students are there in the school district?

Equation: $0.05x = 240$ Solution: $4,800 \text{ students}$

31. The table shows how many miles Uyen has run this week.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Miles	6.5	2.9	4.2	5.5	3.1

The total distance this week is 1.5 times the distance that she ran last week.

How many miles did Uyen run last week? 14.8 mi



Common Core Spiral Review

Solve each equation. Check your solution. 7.EE.4

32. $w + 5 = -20$

-25

33. $x - 17 = -32$

-15

34. $t + 7.2 = 1.65$

-5.55

35. $-0.4 = g - 4.9$

4.5

36. $y - \frac{2}{5} = 1\frac{3}{5}$

2

37. $-5\frac{1}{6} = 2\frac{1}{3} + p$

$-7\frac{1}{2}$

38. **Financial Literacy** Simone saved \$65.35 more than her brother Dan and \$37.50 less than her sister Carly. Carly saved \$127.75. Write and solve equations to find how much money Simone and Dan saved. 6.EE.7

Simone $s + 37.50 = 127.75$ $\$90.25$

Dan $s - 65.35 = d$ $\$24.90$