Lesson 4

Solve Equations with Variables on Each Side

Real-World Link

Watch

Cell Phones A wireless company offers two cell phone plans. Plan A charges $24.95 per month plus $0.10 per minute for calls. Plan B charges $19.95 per month plus $0.20 per minute. Use the questions to find when the two plans cost the same.

Essential Question

WHAT is equaivalence?

ANSWER KEY

Common Core State Standards

Content Standards
8.EE.7, 8.EE.7a, 8.EE.7b

Mathematical Practices
1, 3, 4

1. Complete the table.

Minutes (m)	Plan A $24.95 + 0.10m$	Plan B $19.95 + 0.20m$
10	25.95	21.95
20	26.95	23.95
30	27.95	25.95
40	28.95	27.95
50	29.95	29.95
60	30.95	31.95
70	31.95	33.95

2. For what value(s) does Plan A cost less?
When you use more than 50 minutes

3. For what value(s) does Plan B cost less?
When you use less than 50 minutes

4. For what value(s) do both Plans cost the same?
50 minutes

Which Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Work Zone

Equations with Variables on Each Side

Some equations, like $8 + 4d = 5d$, have variables on each side of the equals sign. To solve, use the properties of equality to write an equivalent equation with the variables on one side of the equals sign. Then solve the equation.

Examples

Tutor

1. **Solve $8 + 4d = 5d$. Check your solution.**

$8 + 4d = 5d$ Write the equation.

$-4d = -4d$ Subtraction Property of Equality

$8 = d$ Simplify by combining like terms.

Subtract $4d$ from the left side of the equation to isolate the variable.

Subtract $4d$ from the right side of the equation to keep it balanced.

To check your solution, replace d with 8 in the original equation.

Check $8 + 4d = 5d$ Write the original equation.

$8 + 4(8) \stackrel{?}{=} 5(8)$ Replace d with 8.

$40 = 40$ ✓ The sentence is true.

2. **Solve $6n - 1 = 4n - 5$.**

$6n - 1 = 4n - 5$ Write the equation.

$-4n \quad = -4n$ Subtraction Property of Equality

$2n - 1 = -5$ Simplify.

$+1 = +1$ Addition Property of Equality

$2n = -4$ Simplify.

$n = -2$ Mentally divide each side by 2.

Check $6n - 1 = 4n - 5$ Write the original equation.

$6(-2) - 1 \stackrel{?}{=} 4(-2) - 5$ Replace n with -2.

$-13 = -13$ ✓ The sentence is true.

Got it? **Do these problems to find out.**

Solve each equation. Check your solution.

a. $8a = 5a + 21$

$-5a \quad -5a$

$\frac{3a}{3} = \frac{21}{3}$

$a = 7$

b. $3x - 7 = 8x + 23$

$-3x \quad -3x$

$-7 = 5x + 23$

$-23 \quad -23$

$\frac{-30}{5} = \frac{5x}{5}$

$x = -6$

Show your work.

a. $a = 7$

b. $x = -6$

Example

Tutor

3. **Green's Gym charges a one time fee of $50 plus $30 per session for a personal trainer. A new fitness center charges a yearly fee of $250 plus $10 for each session with a trainer. For how many sessions is the cost of the two plans the same?**

Words	fee of $50 plus $30 per session	is the same as	a fee of $250 plus $10 per session.
Variable	Let s represent the number of sessions.		
Equation	$50 + 30s = 250 + 10s$		

$50 + 30s = 250 + 10s$	Write the equation.
$-10s = \quad -10s$	Subtraction Property of Equality
$50 + 20s = 250$	Simplify.
$-50 \quad = -50$	Addition Property of Equality
$20s = 200$	Simplify.
$\frac{20s}{20} = \frac{200}{20}$	Division Property of Equality
$s = 10$	Simplify.

So, the cost is the same for 10 personal trainer sessions.

Check

Green's Gym: $50 plus 10 sessions at $30 per session

$50 + 10 \cdot 30 = 50 + 300$

$= \$350$

new fitness center: $250 plus 10 sessions at $10 per session

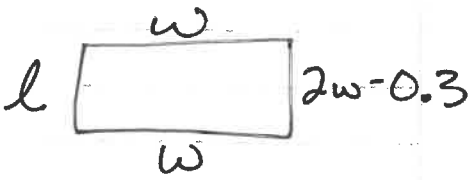
$250 + 10 \cdot 10 = 250 + 100$

$= \$350$ ✓

Got it? **Do this problem to find out.**

c. The length of a flag is 0.3 foot less than twice its width. If the perimeter is 14.4 feet longer than the width, find the dimensions of the flag.

$l = 2w - 0.3$



$P = w + 14.4$

$w + 14.4 = 2(2w - 0.3) + 2w$

$w + 14.4 = 4w - 0.6 + 2w$

$w + 14.4 = 6w - 0.6$

$+0.6 \qquad +0.6$

$w + 15 = 6w$

$-w \qquad -w$

$\frac{15}{5} = \frac{5w}{5}$

$3 = w$

Show your work.

$l = 2(3) - 0.3$

$6 - 0.3 = 5.7$ ft

c. $w = 3$ ft

$l = 5.7$ ft

Equations with Rational Coefficients

In some equations, the coefficients of the variables are rational numbers. Remember when working with fractions, you need to have a common denominator before you add or subtract.

Example

Tutor

4. Solve $\frac{2}{3}x - 1 = 9 - \frac{1}{6}x$.

$\frac{4}{6}x - 1 = 9 - \frac{1}{6}x$	The common denominator of the coefficients is 6. Rewrite the equation.
$+\frac{1}{6}x \quad = \quad +\frac{1}{6}x$	Addition Property of Equality
$\frac{5}{6}x - 1 = 9$	Simplify.
$+1 = +1$	Addition Property of Equality
$\frac{5}{6}x = 10$	Simplify.
$\left(\frac{6}{5}\right)\frac{5}{6}x = 10\left(\frac{6}{5}\right)$	Multiplication Property of Equality
$x = 12$	Simplify.

Show your work.

e. $p = -8$

f. $c = \frac{2}{15}$

Got it? Do these problems to find out.

e. $\frac{1}{2}p + 7 = \frac{3}{4}p + 9$

$-7 \quad -7$

$\frac{1}{2}p = \frac{3}{4}p + 2$

$-\frac{3}{4}p \quad -\frac{3}{4}p$

$-\frac{4}{1} \cdot -\frac{1}{4}p = 2 \cdot -\frac{4}{1}$

$p = -8$

f. $-\frac{5}{4}c - \frac{1}{2} = -\frac{3}{4} + \frac{5}{8}c$

$+\frac{1}{2} \quad +\frac{1}{2}$

$-\frac{5}{4}c = -\frac{1}{4} + \frac{5}{8}c$

$-\frac{5}{8} \quad -\frac{5}{8}$

$-\frac{8}{15} \cdot -\frac{15}{8}c = -\frac{1}{4} \cdot -\frac{8}{15}2$

$-\frac{1}{1} \cdot -\frac{2}{15}$

$c = \frac{2}{15}$

Guided Practice

Check

Solve each equation. Check your solution. (Examples 1, 2, 4)

1. $5n + 9 = 2n$

$-5n \quad -5n$

$\frac{9}{-3} = \frac{-3n}{-3}$

$-3 = n$

Show your work.

2. $7y - 8 = 6y + 1$

$+8 \quad +8$

$7y = 6y + 9$

$-6y \quad -6y$

$y = 9$

3. $\frac{3}{5}x - 15 = \frac{6}{5}x + 12$

$+15 \quad +15$

$\frac{3}{5}x = \frac{6}{5}x + 27$

$-\frac{6}{5} \quad -\frac{6}{5}$

$-\frac{5}{3} \cdot -\frac{3}{5}x = 27 \cdot -\frac{5}{3} = -\frac{45}{1}$

$x = -45$

4. EZ Car Rental charges \$40 a day plus \$0.25 per mile. Ace Rent-A-Car charges \$25 a day plus \$0.45 per mile. What number of miles results in the same cost for one day? (Example 3)

$0.25m + 40 = 0.45m + 25$

$-25 \quad -25$

$0.25m + 15 = 0.45m$

$-0.25m \quad -0.25m$

$\frac{15}{0.20} = \frac{0.20m}{0.20}$

$75 = m$

75 miles

5. **Building on the Essential Question** How is solving an equation with the variable on each side similar to solving a two-step equation?

Rate Yourself!

How well do you understand how to solve equations? Circle the figure that applies.

Clear | Somewhat Clear | Not So Clear

For more help, go online to access a Personal Tutor.

Tutor

Name _____ My Homework _____

Independent Practice

Go online for Step-by-Step Solutions eHelp

Solve each equation. Check your solution. (Examples 1, 2, 4)

1. $7a + 10 = 2a$

$-10 \quad -10$

$7a = 2a - 10$

$-2a \quad -2a$

Show your work.

$\frac{5a}{5} = \frac{-10}{5}$

$a = -2$

2. $11x = 24 + 8x$

$-8x \quad -8x$

$\frac{3x}{3} = \frac{24}{3}$

$x = 8$

3 $8y - 3 = 6y + 17$

$+3 \quad +3$

$8y = 6y + 20$

$-6y \quad -6y$

$\frac{2y}{2} = \frac{20}{2}$

$y = 10$

4. $5p + 2 = 4p - 1$

$+1 \quad +1$

$5p + 3 = 4p$

$-5p \quad -5p$

$\frac{3}{-1} = \frac{-p}{-1}$

$-3 = p$

5. $15 - \frac{1}{6}n = \frac{1}{6}n - 1$

$+\frac{1}{6}n \quad +\frac{1}{6}n$

$15 = \frac{2}{6}n - 1$

$+1 \quad +1$

$\frac{6}{2} \cdot \frac{16^8}{1} = \frac{2}{6}n \cdot \frac{6}{2}$

$n = 48$

6. $3 - \frac{2}{9}b = \frac{3}{9}b - 7$

$+\frac{2}{9}b \quad +\frac{2}{9}b$

$3 = \frac{5}{9}b - 7$

$+7 \quad +7$

$\frac{9}{5} \cdot \frac{10^2}{1} = \frac{5}{9}b \cdot \frac{9}{5}$

$b = 18$

7 Nine fewer than half a number is five more than four times the number. Define a variable, write an equation, and solve to find the number. (Example 3)

$\frac{1}{2}n - 9 = 4n + 5$

$-4n \quad -4n$

$-3\frac{1}{2}n - 9 = 5$

$+9 \quad +9$

$-3\frac{1}{2}n = 14$

$\frac{-3\frac{1}{2}n}{-3\frac{1}{2}} = \frac{14}{-3\frac{1}{2}}$

$n = -4$

8. The table shows ticket prices for the local minor league baseball team for fan club members and non-members. For how many tickets is the cost the same for club members and non-members? (Example 3)

10 tickets

	Ticket Prices	
	Club Members	Non-Club Members
Membership Fee (one-time)	$30	none
Ticket Price	$3	$6

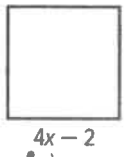
$3t + 30 = 6t$

$-3t \quad -3t$

$30 = 3t$

$10 = t$

9. **Multiple Representations** Refer to the square at the right.



a. **Words** Explain a method you could use to find the value of x.

You know a square has equal side lengths so you can set the equations equal to each other.

b. **Symbols** Write an equation to find the side length of the square.

$4x - 2 = 2x + 8$

c. **Algebra** What is the side length of the square?

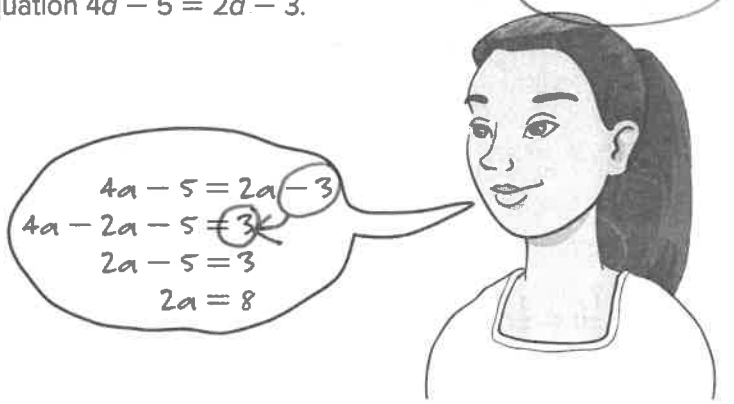
5 units

$4x - 2 = 2x + 8$
$+2 \quad +2$
$4x = 2x + 10$
$-2x \quad -2x$
$\frac{2x}{2} = \frac{10}{2}$
$x = 5$

H.O.T. Problems Higher Order Thinking

10. **Find the Error** Alma is solving the equation $4a - 5 = 2a - 3$. Circle her mistake and correct it.

$4a - 5 = 2a - 3$
$-2a \quad -2a$
$2a - 5 = -3$
$+5 \quad +5$
$\frac{2a}{2} = \frac{2}{2}$
$a = 1$



11. **Model with Mathematics** Write a real-world problem that can be solved using the equation $5x = 3x + 20$. EX: You have 20 crafts made and continue to make three per hour. How many hours will it take you and your friend to make the same amount of crafts if she is making 5 per hour?

12. **Persevere with Problems** Find the area of the rectangle at the right.

147 units^2

$2x + 17 = 6x + 9$
$-2x \quad -2x$
$17 = 4x + 9$
$-9 \quad -9$
$\frac{8}{4} = \frac{4x}{4}$
$x = 2$

$2x + 17$: $7 \times 21 = 147$

$4x - 1$: $4(2) - 1$, $8 - 1$, 7

$6x + 9$: $6(2) + 9$, $12 + 9$, 21

13. **Model with Mathematics** Write two equations so that each have variables on both sides and a solution of $\frac{1}{2}$. Sample: $3x + 6 = x + 7$

Name _____ My Homework _____

Extra Practice

Solve each equation. Check your solution.

14. $9g - 14 = 2g$

Homework Help

$9g - 14 = 2g$
$-9g \quad = -9g$
$\frac{-14}{-7} = \frac{-7g}{-7}$
$2 = g$

15. $-6f + 13 = 2f - 11$
$+11 \quad +11$
$-6f + 24 = 2f$
$+6f \quad +6f$
$\frac{24}{8} = \frac{8f}{8}$
$3 = f$

16. $2.5h - 15 = 4h$
$-2.5h \quad -2.5h$
$\frac{-15}{1.5} = \frac{1.5h}{1.5}$
$-10 = h$

17. $2z - 31 = -9z + 24$
$+31 \quad +31$
$2z = -9z + 55$
$+9z \quad +9z$
$\frac{11z}{11} = \frac{55}{11}$
$z = 5$

18. Will averages 18 points a game and is the all-time scoring leader on his team with 483 points. Tom averages 21 points a game and is currently second on the all-time scorers list with 462 points. If both players continue to play at the same rate, how many more games will it take until Tom and Will have scored the same number of total points?

7 games

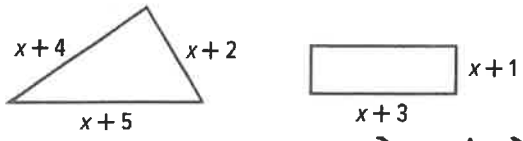
$18g + 483 = 21g + 462$
$-18g \quad -18g$
$483 = 3g + 462$
$-462 \quad -462$
$\frac{21}{3} = \frac{3g}{3}$
$g = 7$

19. Eighteen less than three times a number is twice the number. Define a variable, write an equation, and solve to find the number.

$3n - 18 = 2n$
$+18 \quad +18$
$3n = 2n + 18$
$-2n \quad -2n$
$n = 18$

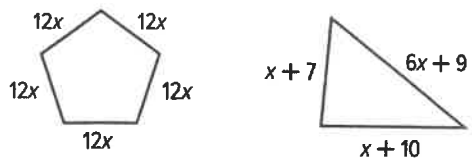
MP Reason Abstractly **Write an equation to find the value of x so that each pair of polygons has the same perimeter. Then solve.**

20.



$x + 4 + x + 5 + x + 2 = 2(x + 3) + 2(x + 1)$
$3x + 11 = 2x + 6 + 2x + 2$
$3x + 11 = 4x + 8$
$-8 \quad -8$
$3x + 3 = 4x$
$-3x \quad -3x$
$3 = x$

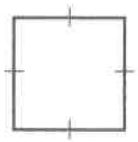
21.



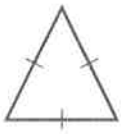
$5(12x) = x + 7 + x + 10 + 6x + 9$
$60x = 8x + 26$
$-8x \quad -8x$
$\frac{52x}{52} = \frac{26}{52}$
$x = 0.5$

Power Up! Common Core Test Practice

22. The two regular polygons below have the same perimeter.

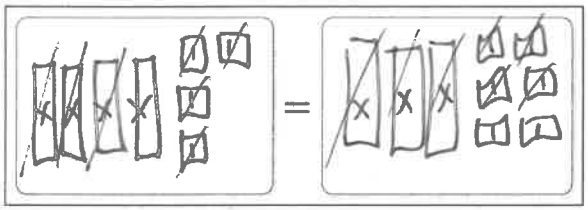


$x+1$



$x+2$

Use the algebra tiles to model an equation on the equation mat below that can be used to find x. Then solve the equation.



$x =$ 2

23. Company A charges \$28.50 plus \$18 a room to clean carpet. Company B charges \$16.50 plus \$20 a room. Determine if each statement is true or false.

a. For 4 rooms, carpet cleaner B is cheaper. ● True ○ False

b. For 5 rooms, carpet cleaner A is cheaper. ○ True ● False

c. The equation $28.5 + 18x = 16.5 + 20x$ can be solved to find the number of rooms for which the total cost is the same. ● True ○ False

d. For 6 rooms, both carpet cleaners charge the same amount. ● True ○ False

Qc&d

A B

$18r + 28.5 = 20r + 16.5$

$-16.5 \quad -16.5$

$18r + 12 = 20r$

$-18r \quad -18r$

$12 = 2r$

$6 = r$

Qa

(A) $18(4) + 28.50$

100.50

(B) $20(4) + 16.50$

96.50

Qb

A $18(5) + 28.50$

$90 + 28.50$

118.50

B $20(5) + 16.50$

116.50

Common Core Spiral Review

Use the Distributive Property to write each expression as an equivalent expression. 7.EE.1

24. $6(x + 5) =$ $6x + 30$

25. $-8(y - 1) =$ $-8y + 8$

26. $-3(-5z + 12) =$ $15z - 36$

27. $\frac{1}{3}(6z + 10) =$ $2z + \frac{10}{3}$